Statistics

Lecture 14

Feb 19-8:47 AM

(SG 16)

Binomial Prob. dist

1) n independent events

2) $P(\text{Success}) = p$, $P(\text{Failure}) = q$

$p + q = 1$, $q = 1 - p$

p & q do not change for different events

3) $x \to$ # of Successes , $n - x \to$ # of failures

$$P(x) = {}_nC_x \cdot p^x \cdot q^{n-x}$$

ex: flip a fair coin 100 times. $n = 100$

Success is to land tails $p = .5$, $q = .5$

P(land $\underbrace{\text{exactly}}$ 55 tails)

$$P(x=55) = \underset{n}{100}C_{\underset{x}{55}} \cdot \underset{p}{(.5)}^{\overset{x}{55}} \cdot \underset{q}{(.5)}^{\overset{n-x}{45}} = \boxed{.048}$$

$\boxed{\text{2nd}}$

$\boxed{\text{VARS}}$

$\downarrow$

binompdf$(\overset{n}{100}, \overset{p}{.5}, \overset{x}{55}) = \boxed{.048}$

Apr 2-1:47 PM

You are taking multiple choice exam with 20 questions.

Each question has 4 choices but only one correct choice

$n = 20$

$p = \frac{1}{4} = .25$

You are making random guesses. $q = \frac{3}{4} = .75$

P(Corretly guess 8 correct answers) =

$P(x = 8) = {}_{20}C_{8} \cdot (.25)^{8} \cdot (.75)^{12} = \boxed{.061}$

$n \quad x \quad p \quad q$

$x \quad n - x$

$P(x = 8) = \text{binompdf}(20, .25, 8) = \boxed{.061}$

Apr 2-1:56 PM

A quiz has 20 True-False questions.

$n = 20$

You are making random guesses. $p = .5$

$q = .5$

P(exactly 12 correct ans)

$P(x = 12) = \text{binompdf}(20, .5, 12) = \boxed{.120}$

P(at most 12 correct ans)

$P(x \leq 12) = \text{binomcdf}(20, .5, 12) = \boxed{.868}$

P(atleast 12 correct ans) =

Total Prob.

$P(x \geq 12) = 1 - P(x \leq 11)$

we don't want 11 | we want 12

$= 1 - \text{binomcdf}(20, .5, 11) = \boxed{.252}$

Apr 2-2:06 PM

Consider a binomial prob. dist with $n=400$ and $p=.8$.

1) $q=1-p=\boxed{.2}$

2) $np=400(.8)=\boxed{320}$

3) $npq=400(.8)(.2)=\boxed{64}$

4) $\sqrt{npq}=\sqrt{64}=\boxed{8}$

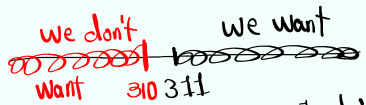
5) $P(x=310)=\text{binompdf}(400,.8,310)=\boxed{.022}$

6) $P(x<310)=P(x\le 309)$

$=\text{binomcdf}(400,.8,309)\approx\boxed{.096}$

7) $P(x>310)=P(x\ge 311)=1-P(x\le 310)$

Total Prob.



$=1-\text{binomcdf}(400,.8,310)$

$=\boxed{.882}$

Apr 2-2:15 PM

Consider a binomial prob. dist. with $n=400$ and $p=.5$

1) $q=1-p$
$=\boxed{.5}$

2) $np=400(.5)$
$=\boxed{200}$

3) $npq=400(.5)(.5)$
$=\boxed{100}$

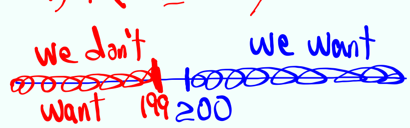
4) $\sqrt{npq}=\sqrt{100}=\boxed{10}$

5) $P(x=205)=\text{binompdf}(400,.5,205)=\boxed{.035}$

6) $P(x\le 220)=\text{binomcdf}(400,.5,220)=\boxed{.980}$

7) $P(x\ge 200)=1-P(x\le 199)$

$=1-\text{binomcdf}(400,.5,199)$



$=\boxed{.520}$

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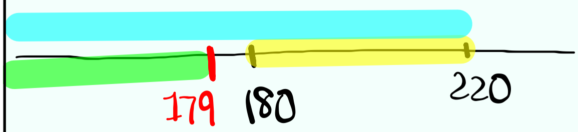
8) $P(180 \le x \le 220) = P(x \le 220) - P(x \le 179)$



$= \text{binomcdf}(400, .5, 220)$

$- \text{binomcdf}(400, .5, 179)$

$= \boxed{.960}$

Apr 2-2:36 PM

You are taking an exam with 100 questions.

Each question has 5 choices but only one correct choice.

Making random guesses.

1) $n = 100$ 2) $p = \frac{1}{5} = .2$ 3) $q = \frac{4}{5} = .8$

4) $np = 100(.2) = \boxed{20}$ 5) $npq = 100(.2)(.8) = \boxed{16}$ 6) $\sqrt{npq} = \sqrt{16} = \boxed{4}$

7) P(guess exactly 25 correct answers)

$P(x=25) = \text{binompdf}(100, .2, 25) = \boxed{.044}$

8) P(guess fewer than 25 correct answers)

$P(x<25) = P(x \le 24)$

$= \text{binomcdf}(100, .2, 24) = \boxed{.869}$

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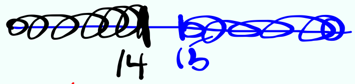
9) P(guess at least 15 Correct answers)

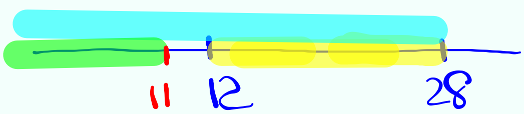
$P(x \geq 15) = 1 - \text{binomcdf}(100, .2, 14) = \boxed{.920}$



10) P(guess between 12 and 28 Correct answers, inclusive)

$P(12 \leq x \leq 28) = \text{binomcdf}(100, .2, 28)$

$- \text{binomcdf}(100, .2, 11)$



$= \boxed{.967}$

Apr 2-2:53 PM

Mean μ

Variance σ^2

Standard Deviation σ

$$\boxed{\begin{aligned} \mu &= np \\ \sigma^2 &= npq \\ \sigma &= \sqrt{\sigma^2} \end{aligned}}$$

Binomial Prob. dist.

Apr 2-3:00 PM

100 Newborn babies are randomly Selected

Success is to have a boy.

1) $n = 100$ 2) $p = .5$ 3) $q = .5$

4) $\mu = np = \boxed{50}$ 5) $\sigma^2 = npq = \boxed{25}$ 6) $\sigma = \sqrt{\sigma^2} = \sqrt{25} = \boxed{5}$

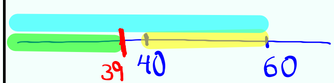
7) 68% Range $\Rightarrow \mu \pm \sigma = 50 \pm 5 \Rightarrow \boxed{45 \text{ to } 55}$

8) usual Range $\Rightarrow \mu \pm 2\sigma = 50 \pm 2(5) \Rightarrow \boxed{40 \text{ to } 60}$

95% Range

9) P($\underbrace{\text{\# of boys}}_{x}$ is $\underbrace{\text{between 40 and 60, inclusive}}_{40 \le x \le 60}$)

$P(40 \le x \le 60) = P(x \le 60) - P(x \le 39)$



$\Rightarrow \boxed{.965}$

$= \text{binomcdf}(100, .5, 60) - \text{binomcdf}(100, .5, 39)$

Apr 2-3:03 PM

I flipped a loaded coin 60 times.

Success is to land tails.

$P(\text{Tails}) = .4$

1) $n = \boxed{60}$ 2) $p = \boxed{.4}$ 3) $q = \boxed{.6}$

4) $\mu = np = \boxed{24}$ 5) $\sigma^2 = npq = \boxed{14.4}$ 6) $\sigma = \sqrt{\sigma^2}$

4

$\sigma = \sqrt{14.4}$

Round μ & σ to whole #

≈ 3.795

7) 68% Range $\mu \pm \sigma \Rightarrow \boxed{20 \text{ to } 28}$

8) 95% Range $\mu \pm 2\sigma \Rightarrow \boxed{16 \text{ to } 32}$

usual Range

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Consider a binomial prob. dist. with $n = 45$ & $p = \frac{1}{3}$.

1) $q = 1 - p = \boxed{\frac{2}{3}}$

2) $\mu = np = 45\left(\frac{1}{3}\right) = \boxed{15}$

3) $\sigma^2 = npq = 45\left(\frac{1}{3}\right)\left(\frac{2}{3}\right) = \boxed{10}$

4) $\sigma = \sqrt{\sigma^2} = \sqrt{10} \approx \boxed{3}$

5) 99.7% Range $\mu \pm 3\sigma = 15 \pm 3(3) = \boxed{6 \text{ to } 24}$

10) $P(6 \le x \le 24) =$

binomcdf(45, 1/3, 24) − binomcdf(45, 1/3, 5)

$= .998 \approx \boxed{99.8\%}$

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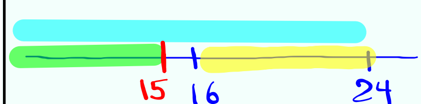
9) P($\underbrace{\text{\# of tails}}_{x}$ is $\underbrace{\text{between 16 to 24}}_{16 \le x \le 24}$, inclusive)

$P(16 \le x \le 24) =$ binomcdf(60, .4, 24) − binomcdf(60, .4, 15)



$= \boxed{.545}$

10) P(# tails is between 20 & 28, inclusive)

$P(20 \le x \le 28)$

= binomcdf(60, .4, 28) − binomcdf(60, .4, 19)

$= \boxed{.765}$

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